Approximate Quantum Circuit Synthesis using Block Encodings

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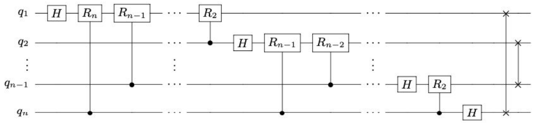
Scalable Solvers Group

2021 CS Area Postdoc Symposium Presentation

The challenge and promise of quantum computing

Image: Berkeley Lab



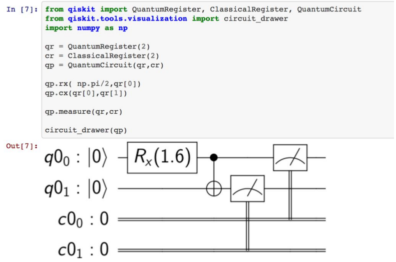


Image: Qiskit Tutorial

Quantum hardware

Quantum algorithms

Software stack

Promise: ***Quantum speedups for some classically intractable problems***

This talk:

New solution to the *quantum circuit synthesis* problem leveraging matrix and tensor decompositions and using *block encodings*

Tensor product structure of quantum states

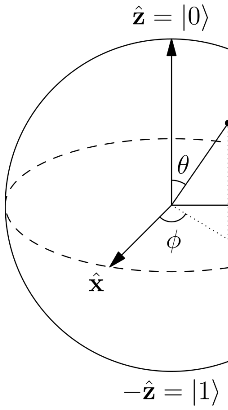
1 qubit	n qubits

1 qubit

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}$$

$$|1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$$

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle = \begin{bmatrix}\alpha\\\beta\end{bmatrix}$$



State space: $\mathcal{H}^2$

n qubits

$$|00\cdots0\rangle = \begin{bmatrix}1\\0\end{bmatrix} \otimes \begin{bmatrix}1\\0\end{bmatrix} \otimes\cdots\otimes \begin{bmatrix}1\\0\end{bmatrix}$$

$$|00\cdots1\rangle = \begin{bmatrix}1\\0\end{bmatrix} \otimes \begin{bmatrix}1\\0\end{bmatrix} \otimes\cdots\otimes \begin{bmatrix}0\\1\end{bmatrix}$$

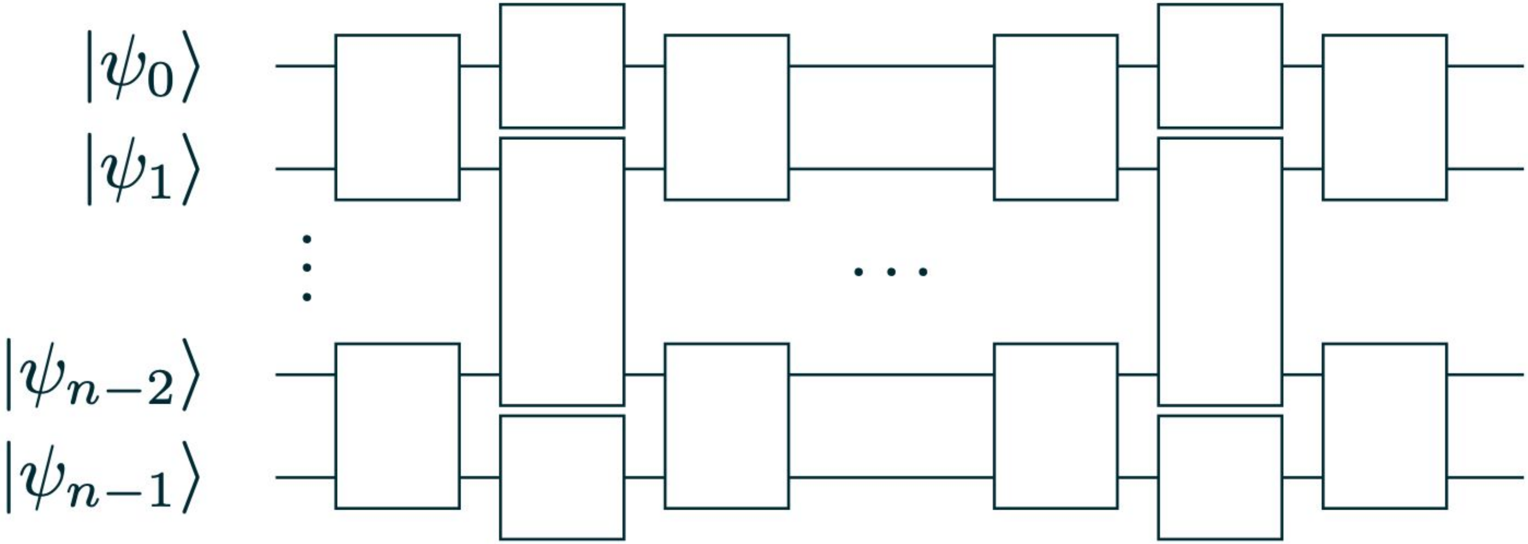
$$\vdots$$

$$|11\cdots1\rangle = \begin{bmatrix}0\\1\end{bmatrix} \otimes \begin{bmatrix}0\\1\end{bmatrix} \otimes\cdots\otimes \begin{bmatrix}0\\1\end{bmatrix}$$

$$|\psi\rangle = \alpha_0\,|00\cdots0\rangle + \alpha_1\,|00\cdots1\rangle + \cdots + \alpha_{2^n-1}\,|11\cdots1\rangle = \begin{bmatrix}\alpha_0\\\alpha_1\\\vdots\\\alpha_{2^n-1}\end{bmatrix}$$

State space: $\mathcal{H}^2 \otimes \mathcal{H}^2 \otimes\cdots\otimes \mathcal{H}^2 = (\mathcal{H}^2)^{\otimes n} = \mathcal{H}^{2^n}$

Quantum algorithms: unitaries with efficient quantum circuits



Synthesis: A well studied subject with many different approaches

Algebraic Techniques

Cosine-Sine Decomposition

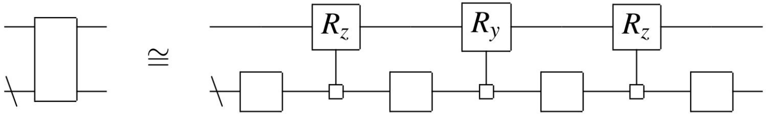


Image: Shende, Bullock, Markov (2006)

KAK Decomposition

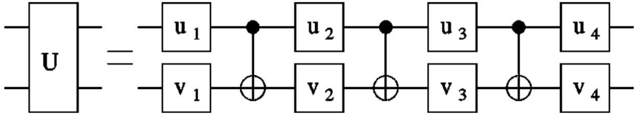


Image: Vidal, Dawson (2004)

Givens QR Decomposition

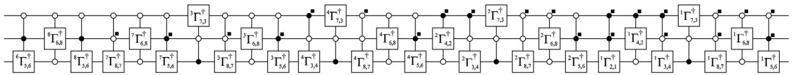


Image: Vartiainen, Mötiönen, Salomaa (2004)

Optimization Techniques

QFAST

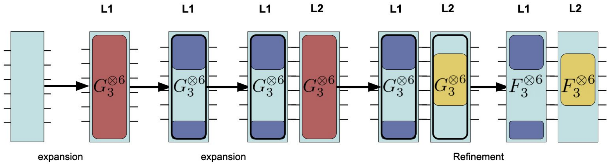


Image: Younis, Sen, Yelick, Iancu (2020)

Repeat-Until-Success Techniques

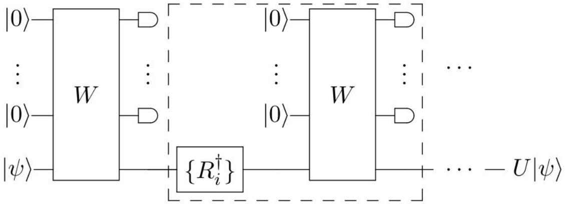
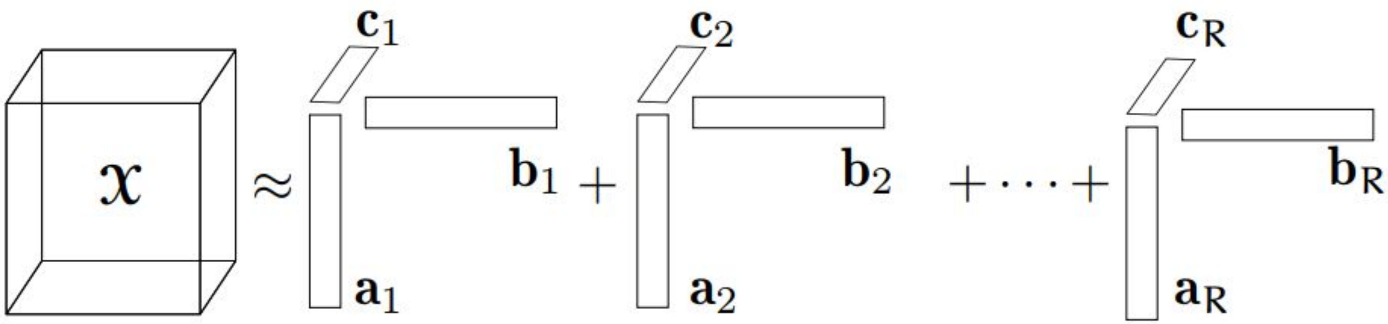


Image: Paetznick, Svore (2014)

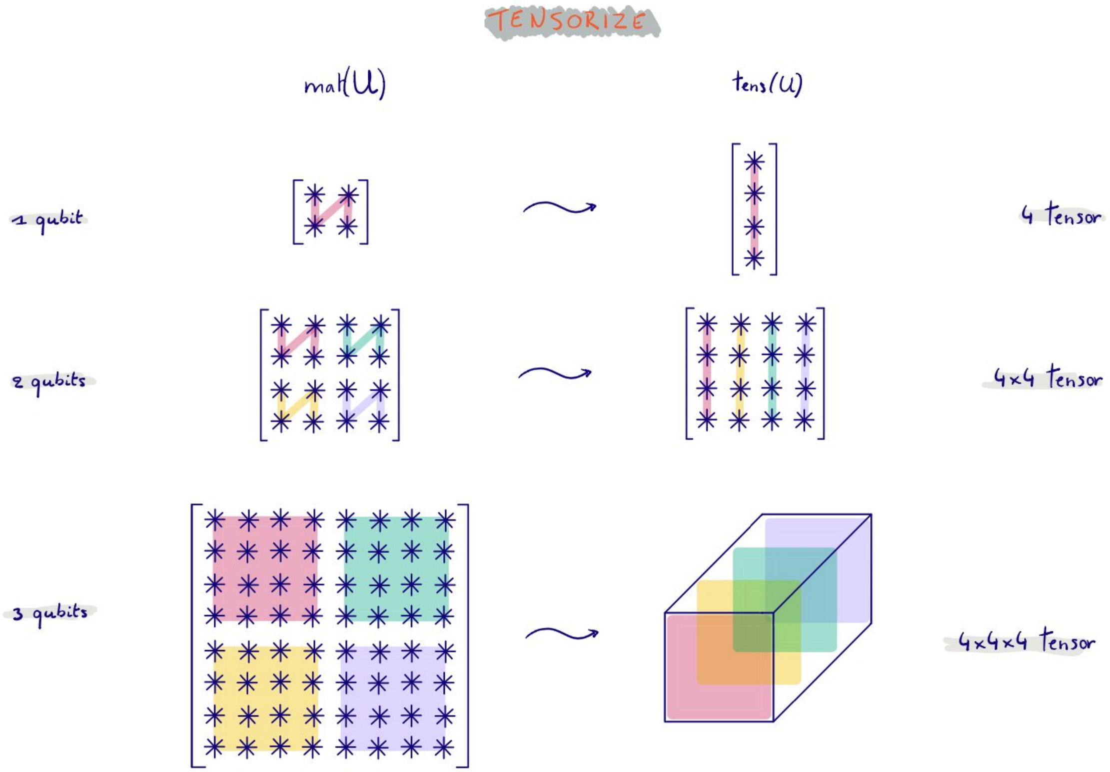
Computational tool from numerical linear algebra

Tensor Rank Decomposition



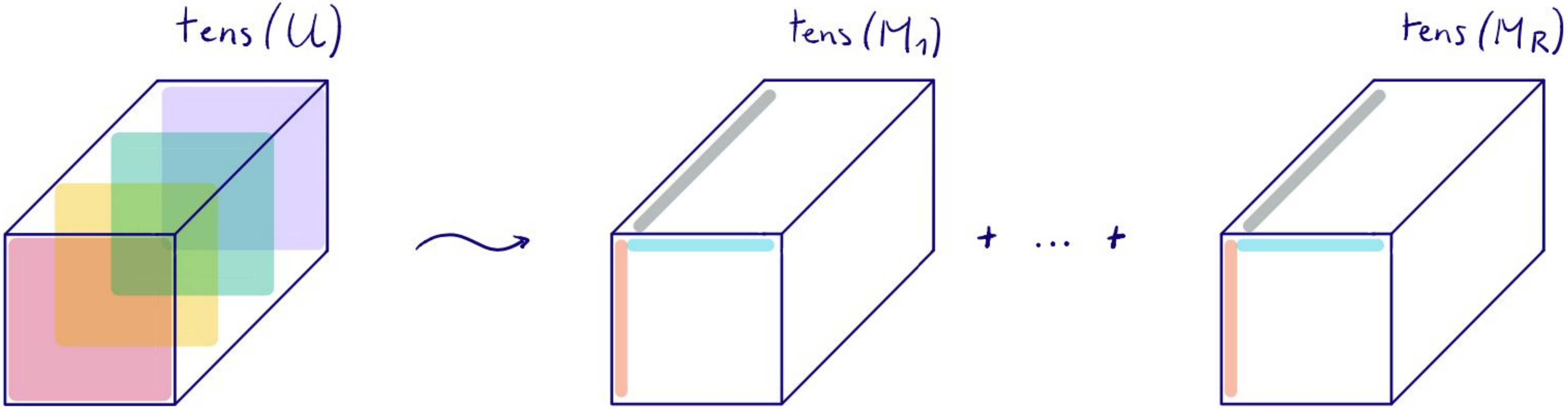
- Widely used in:
  - numerical linear algebra
  - scientific computing
  - data analysis
- Uniqueness
- Good optimization algorithms

Tensorizing the unitary

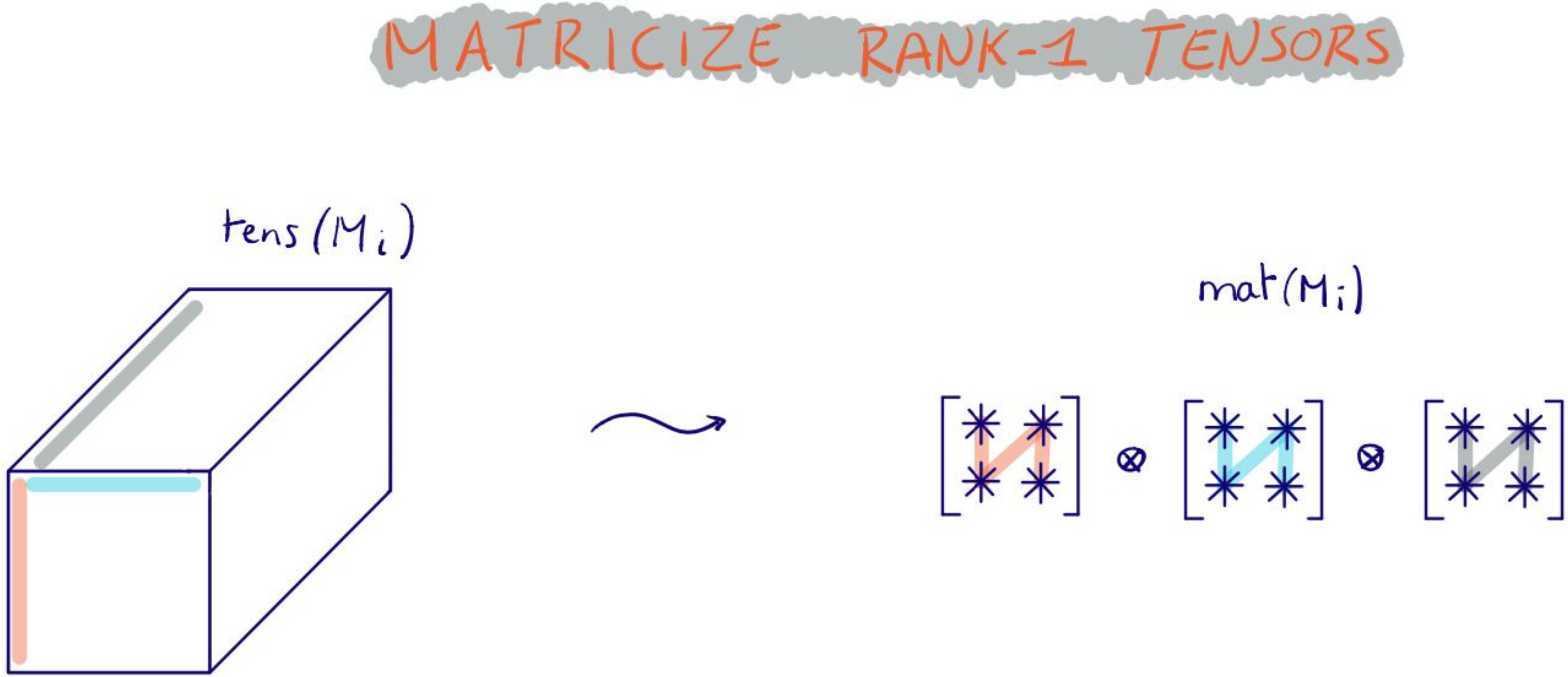


Decompose the tensor

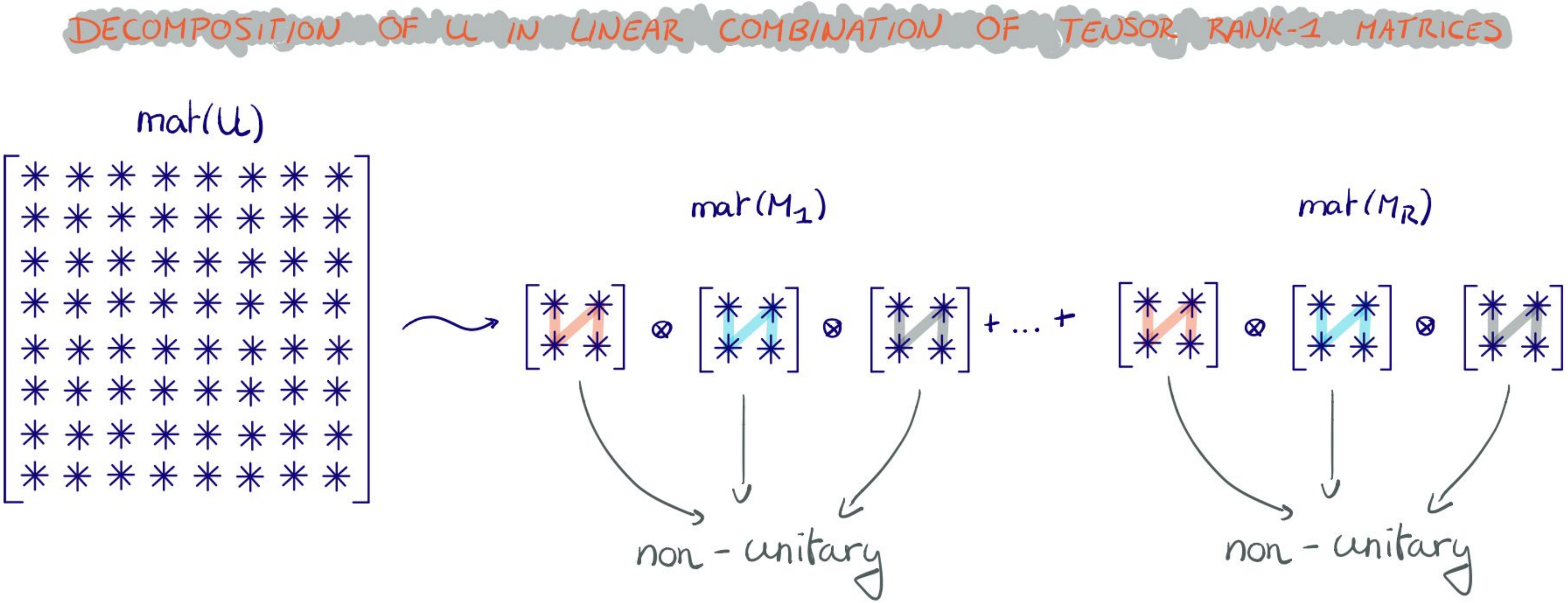
DECOMPOSE TENSOR IN RANK-1 TERMS



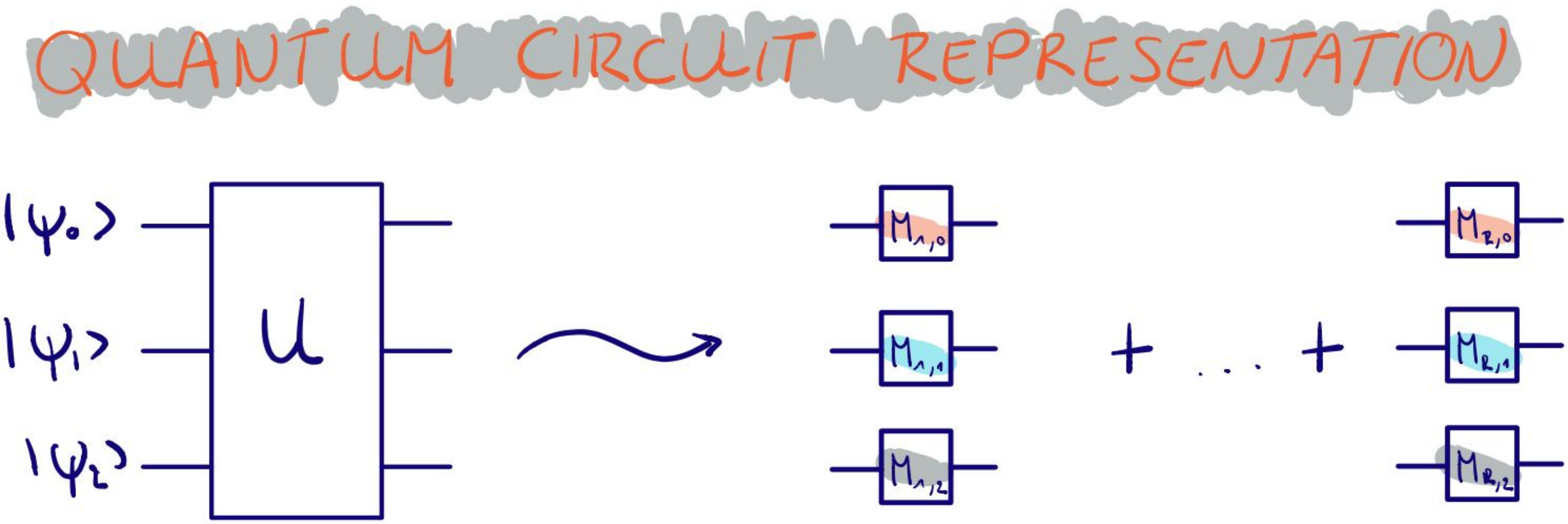
Matricizing the rank-1 tensors



Unitary decomposed in tensor rank-1 matrices

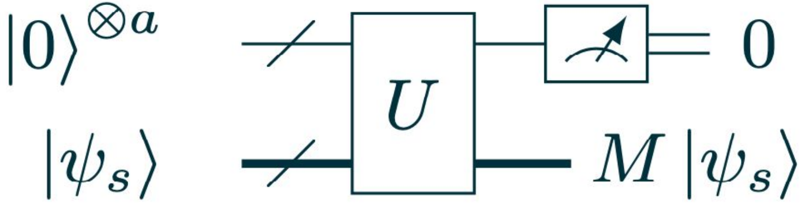


More or less a quantum circuit diagram



Relaxing unitary constraints using block encodings

$$M \longrightarrow U := \begin{bmatrix} M & * \\ * & * \end{bmatrix} \longrightarrow$$



- Embed the non-unitary matrix M in a larger unitary matrix U
- Distinction between additional *ancilla* qubits and *signal* qubits
- Measurement of *ancilla* qubits
  - Probabilistic implementation of M, similar to Repeat-Until-Success strategy
- Amplitude amplification

Combining block encodings in tensor products

$$M^{(1)}$$

$$\downarrow$$

$$\begin{bmatrix} M^{(1)} & * \\ * & * \end{bmatrix}$$

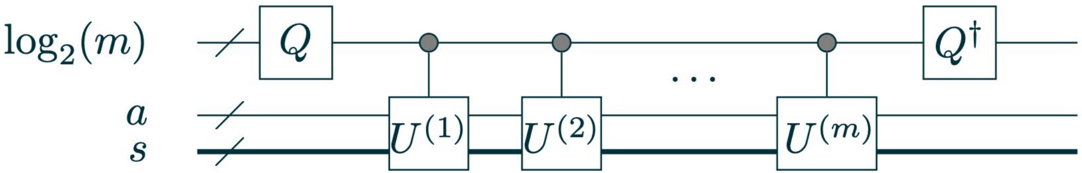
$$\downarrow$$

$$U^{(1)}$$

Sums of block encodings

$$y_1 M^{(1)} \quad + \quad y_2 M^{(2)} \quad + \quad \cdots \quad + \quad y_m M^{(m)}$$

$$U^{(i)} = \begin{bmatrix} M^{(i)} & * \\ * & * \end{bmatrix}$$

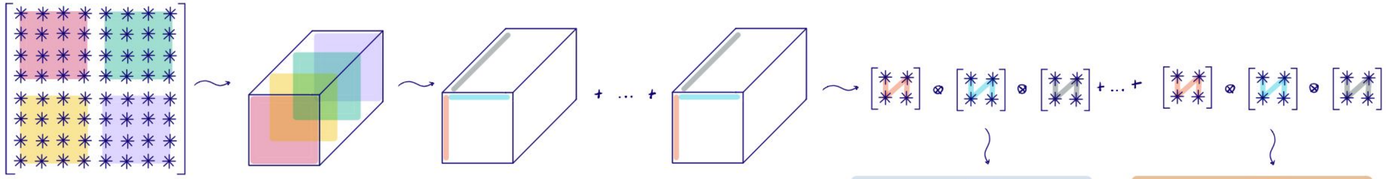


$$Q = \frac{1}{\|y\|_1} \begin{bmatrix} \sqrt{y_1} & * & \cdots & * \\ \sqrt{y_2} & * & \cdots & * \\ \vdots & \vdots & \ddots & \vdots \\ \sqrt{y_m} & * & \cdots & * \end{bmatrix}$$

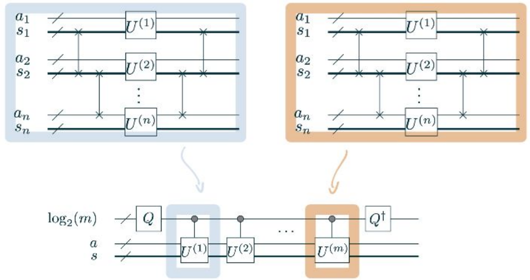
$$\downarrow$$

$$U = \begin{bmatrix} y_1 M^{(1)} + y_2 M^{(2)} + \cdots + y_m M^{(m)} & * \\ * & * \end{bmatrix}$$

Bringing it all together



$$U = \begin{bmatrix} \sum_{i=1}^{m} y_i M^{(i,1)} \otimes M^{(i,2)} \otimes \cdots \otimes M^{(i,n_i)} & * \\ * & * \end{bmatrix}$$



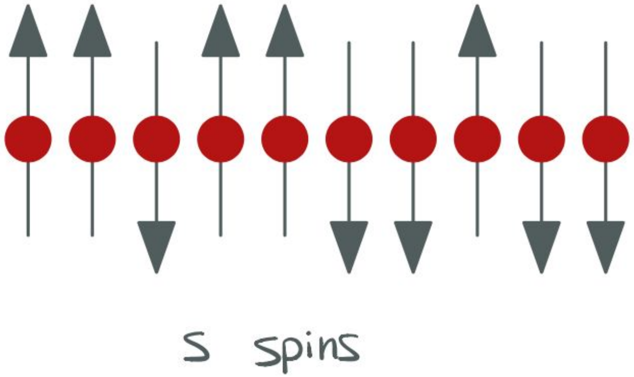
This circuit construction has a sub-exponential efficient gate complexity if:

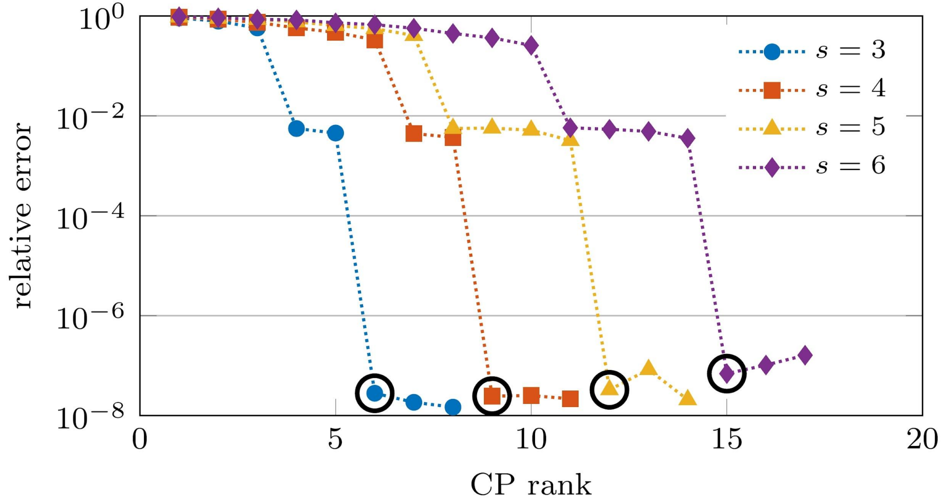
- the tensor rank is sub-exponential
- efficient circuits exist for the individual block encodings

Localized Hamiltonians have low-rank tensor structure

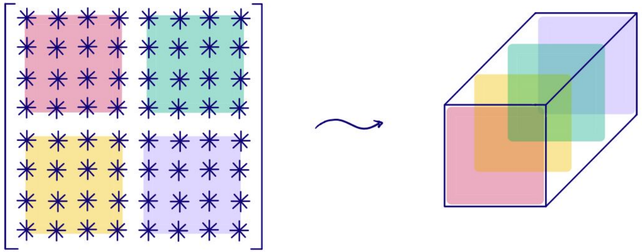
Heisenberg XYZ Hamiltonian

$$H_{\mathrm{XYZ}} = \sum_{i=1}^{s-1} X^{(i)} X^{(i+1)} + Y^{(i)} Y^{(i+1)} + Z^{(i)} Z^{(i+1)}$$





Conclusion



- Block encodings can be easily combined through:
  - Tensor/Kronecker products
  - Linear combinations
- Low-rank tensor decompositions lead to efficient quantum circuits that scale well
- Many problems naturally have (approximate) low-rank tensor structure
  - Localized Hamiltonians
  - Discretized differential operators

Reference: Camps D. and Van Beeumen R., *Approximate quantum circuit synthesis using block encodings*, Phys. Rev. A 102, 052411. DOI:10.1103/PhysRevA.102.052411. arXiv:2007.01417.