



# A Performance Model for Estimating the Cost of Scaling to Practical Quantum Advantage

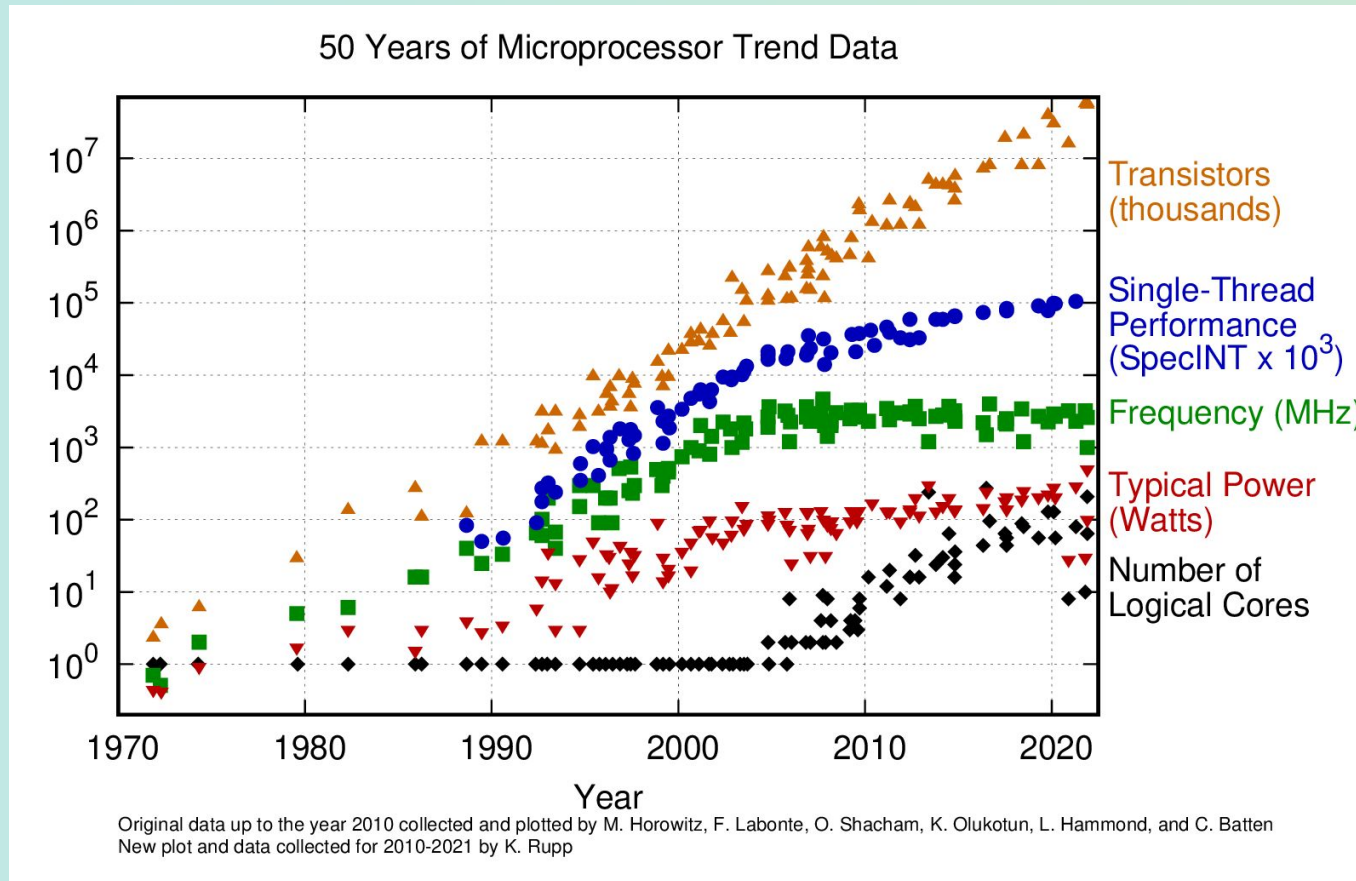
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# Economical and physical limits to Moore's scaling



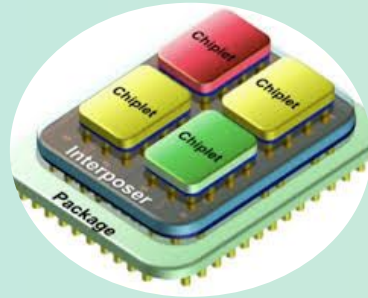
Signs of slowdown in Moore's scaling:

- cost per transistor flattening off?
- cost per fab going up?
- diminishing returns in process size improvements?
- ...

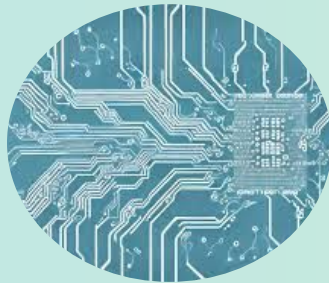
# Opportunities for new approaches to computing



FPGA

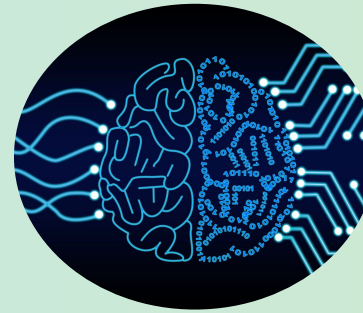


Chiplet



ASIC

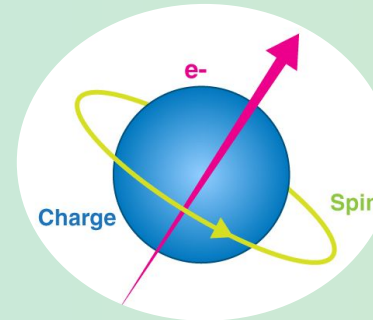
Improved energy efficiency  
through hardware specialization



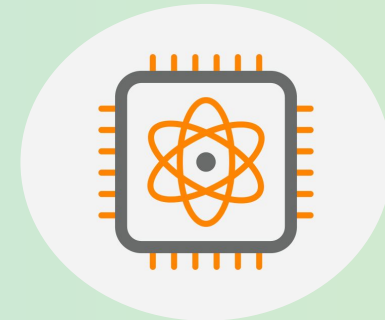
Neuromorphic



Stochastic



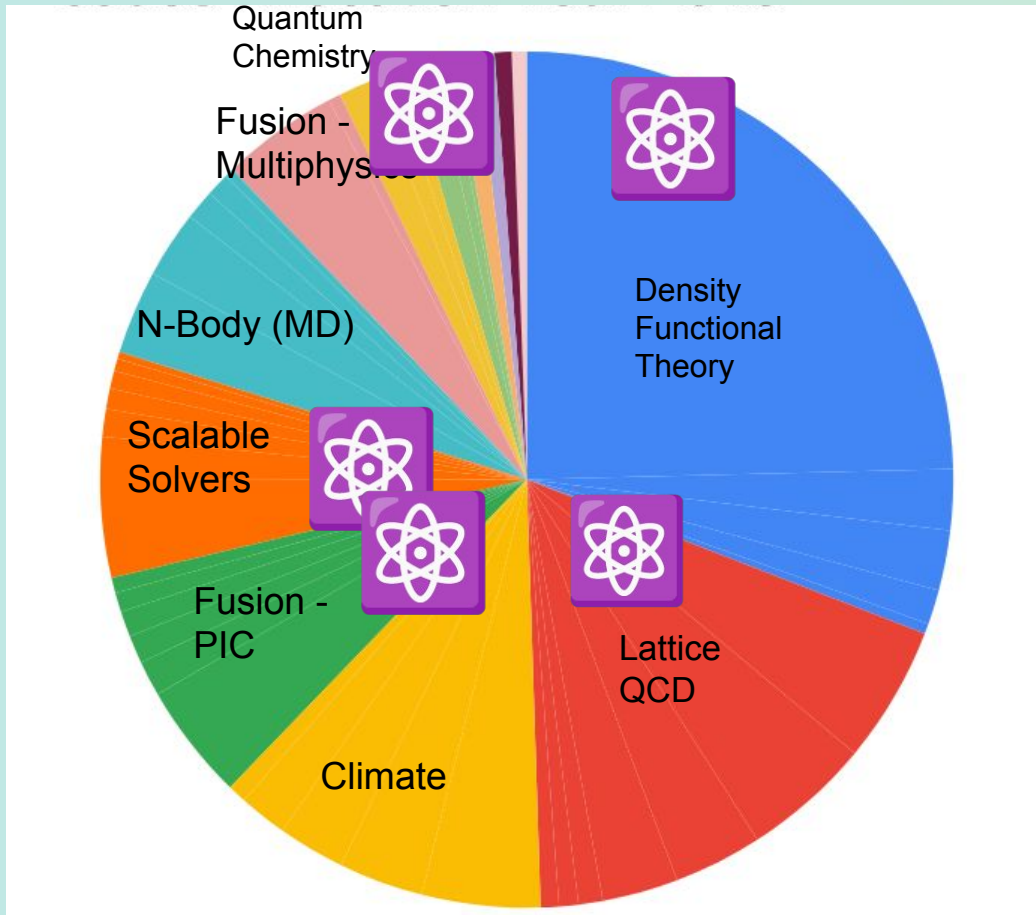
Spintronics





Quantum

New paradigms for computing

# Why quantum computing for HPC?

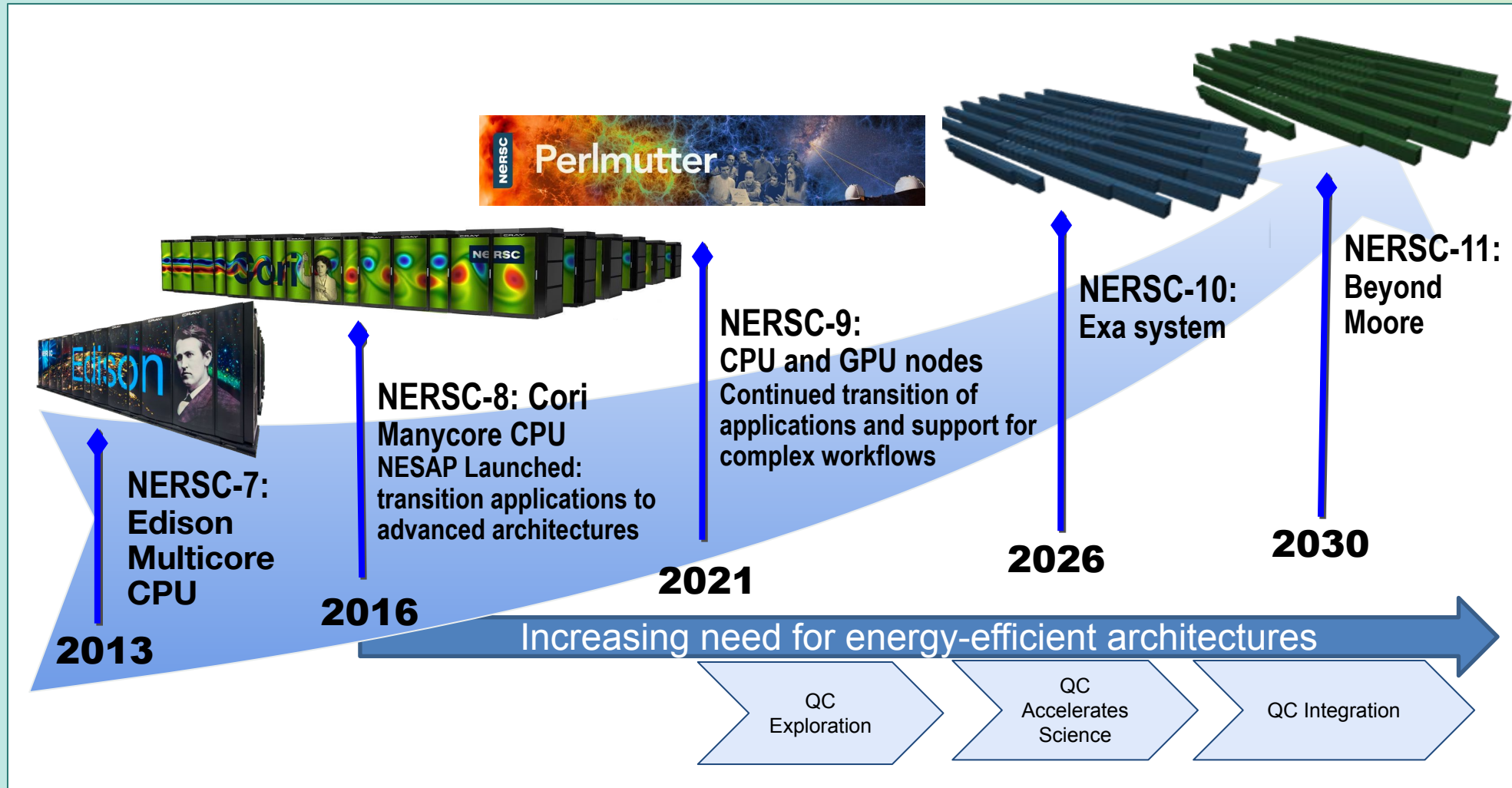


 Quantum mechanical problem  
Quantum algorithms proposed  
**>50% of compute cycles**

 non-Quantum mechanical problem  
Quantum algorithms proposed  
**20% of cycles**


What is **not** on the pie chart? Enabling the previously inconceivable with quantum technologies?

# When quantum computing for HPC?





# How much will quantum computation cost?



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8 logical qubits [Gidney, Craig, et al. "How to factor 2048 bit RSA integers in 8 hours using 20 million noisy qubits."]. This totals 432 billion qubit-seconds. (\$21.6B)

**Precisely worked out examples include:**

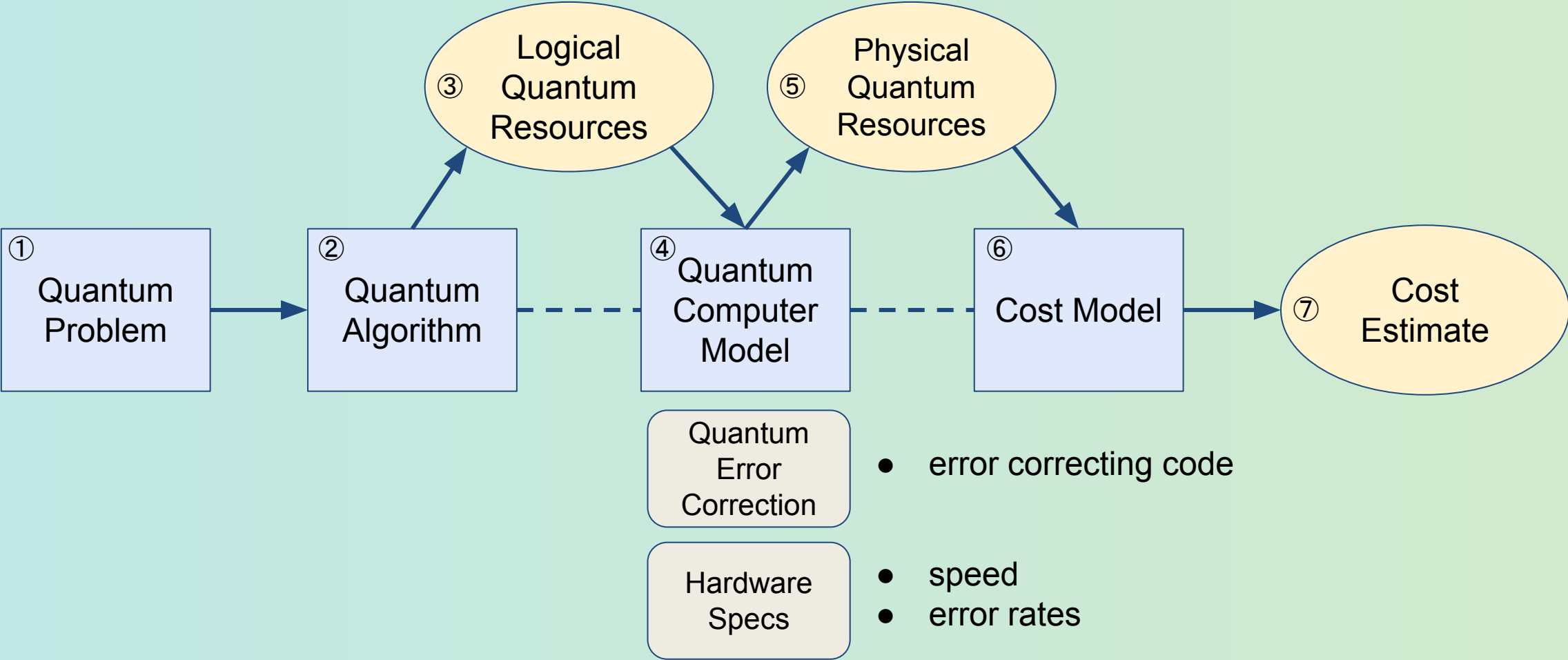
- The main enzyme responsible for drug metabolism, cytochrome P450 (CYP), has unknown mechanisms of promoting oxidation, and simulating it will require 7.8 billion operations in 1434 logical qubits [Goings, Joshua J., et al. "Reliably assessing the electronic structure of cytochrome P450 on today's classical computers and tomorrow's quantum computers"]. In order to perform this many operations, each logical qubit will need to contain approximately 9745 physical qubits. **This totals 109 billion qubit-seconds. (\$5.45B)**
- The mechanism for biological fixation of nitrogen is a coveted chemical process that could significantly reduce the price and energy consumption of production of fertilizers, and it is based on the FeMo cofactor in a yet-not-understood process. This process can be simulated in a quantum computer with 2196 logical qubits in 32 billion operations [Lee, Joonho, et al. "Even more efficient quantum computations of chemistry through tensor hypercontraction".] **This totals 448 billion qubit-seconds. (\$22.4B)**
- In order to factor a 2048 bit product of two primes, a quantum computer will require approximately 25 billion operations in 14238 logical qubits [Gidney, Craig, and Martin Ekerå. "How to factor 2048 bit RSA integers in 8 hours using 20 million noisy qubits."]. **This totals 432 billion qubit-seconds. (\$21.6B)**

**\$5B-\$20B**

All of these are very hard, very large problems!

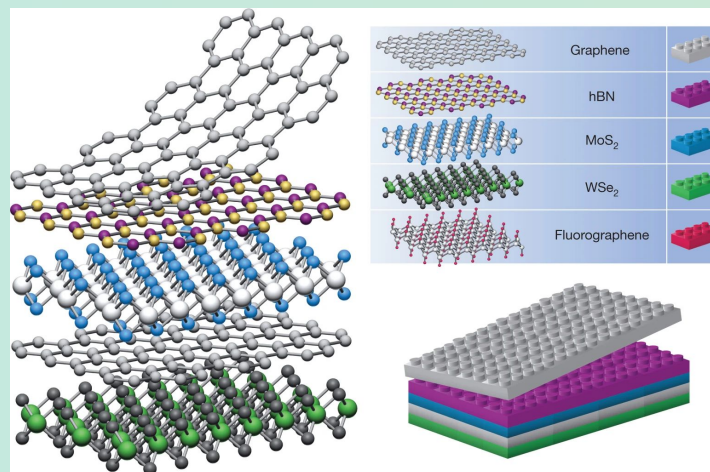
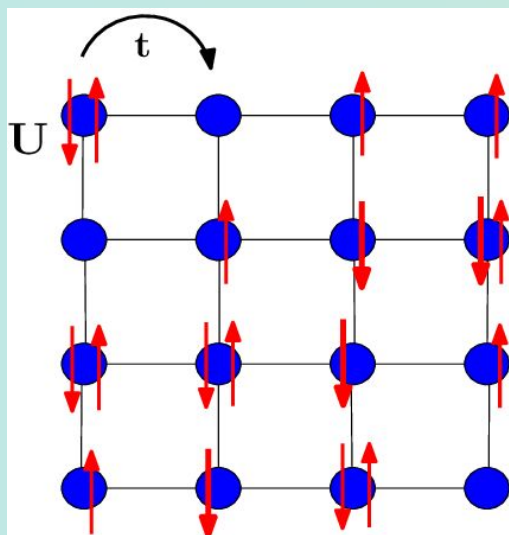
How much would it cost to run the first scientifically relevant, classically-intractable calculation?

# Flowchart for quantum computing cost estimation



# Two-dimensional Hubbard model

- Quantum physics model of interacting **electrons on a square lattice with  $L^2$  sites**
- Complex phenomena such as **metal-insulator transitions** and **superconductivity**
- Applications in 2D materials
- Good approximate classical algorithms exist based on tensor networks and quantum monte carlo
  - They break down in certain difficult regimes that are of most interest
- General solution is classically hard and scales exponentially



Source: Geim, *Nature* (499) 419–425

## Classical complexity:

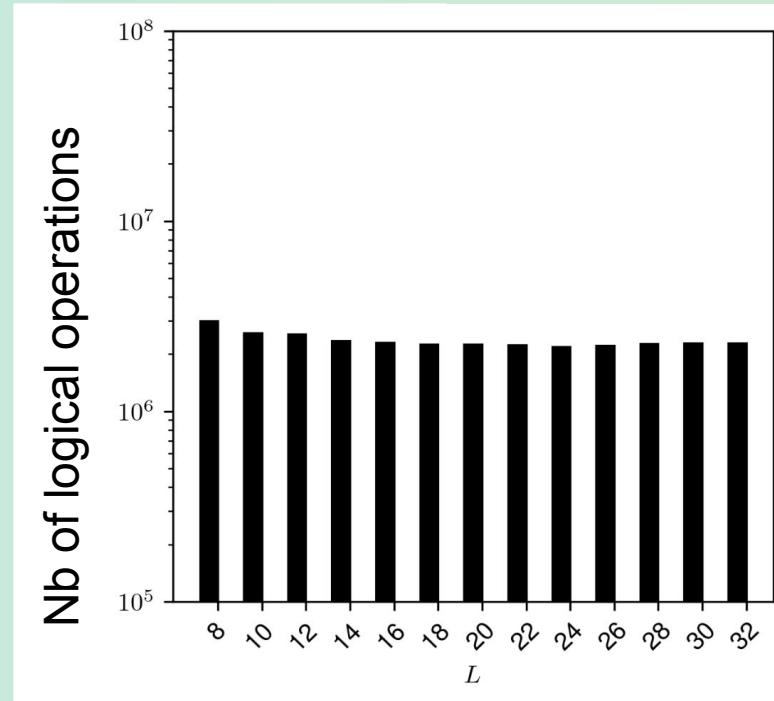
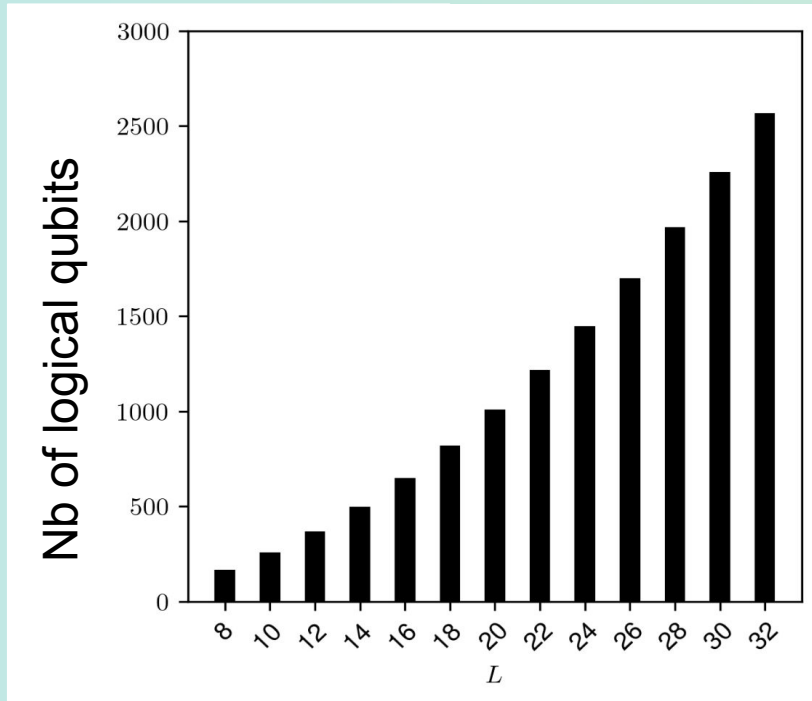
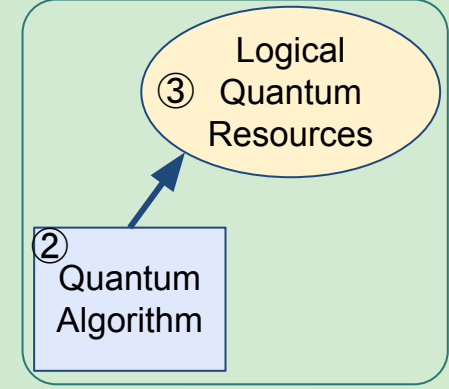
- exact diagonalization - memory:  $\mathcal{O}(4^{L^2})$
- Quantum Monte Carlo: 13h on 3,456 CPUs (Fugaku) in “easy regime” for 8 x 8 lattice

*Charlebois, Imada, PRX (2020)*



# Logical Quantum Resources for $L^2$ Hubbard Model

- **Quantum phase estimation** for computing ground state energy in “hard regime”
- $O(L^2)$  scaling of quantum algorithm → **exponential advantage** over  $\mathcal{O}(4^{L^2})$  classical scaling
- Fixed relative error per lattice site leads to constant logical operations
- Perfect weak scaling



**Logical Resources**

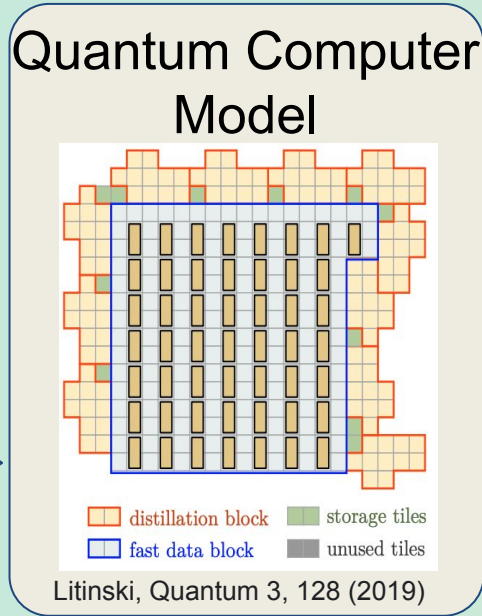
- Logical qubits
- Logical gates



# Convert Logical to Physical Quantum Resources

**Logical Resources**

- Logical qubits
- Logical gates



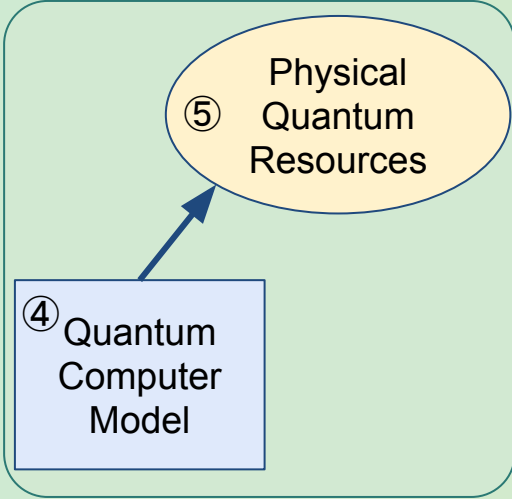
**Physical Resources**

- Physical qubits
- Run time

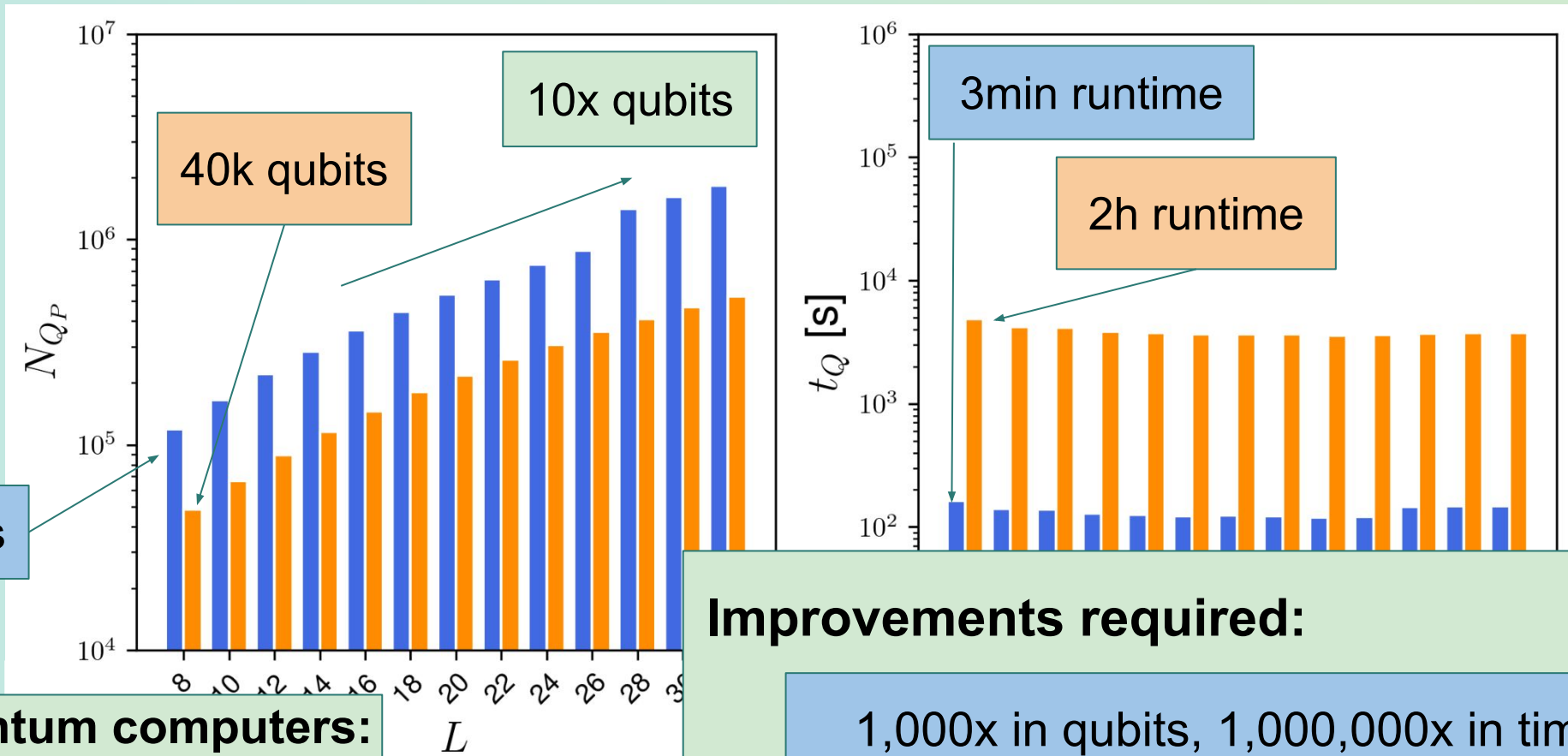
technology	rel. speed	rel. error
superconducting QC	200x	10x
ion trap QC	1x	1x

**Figure of merit:**  $N_{QP} \cdot t_Q$

units: *qubit-seconds*  
(similar to *node hours*)



# Physical Resources for the Hubbard Model



## Today's quantum computers:

100 qubits, max  $10^{-4}$ s

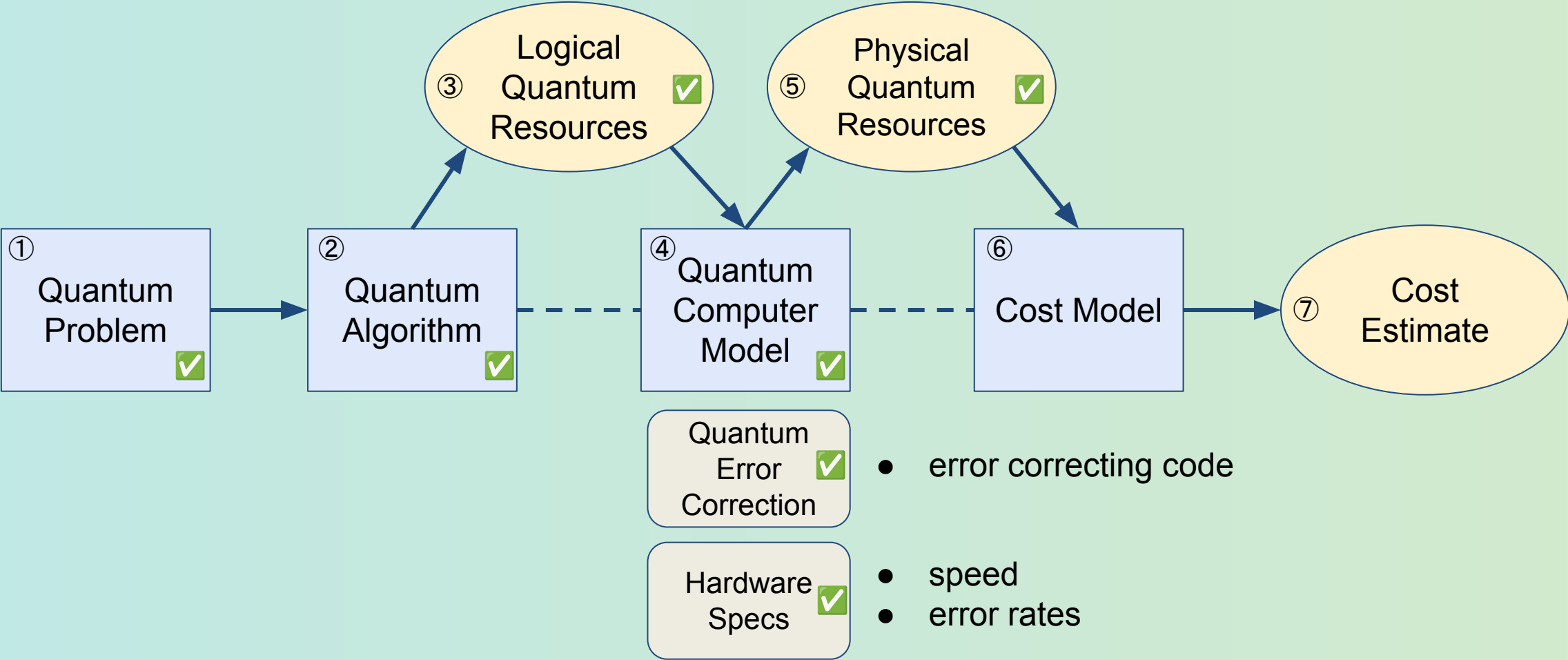
20 qubits, max 1s

## Improvements required:

1,000x in qubits, 1,000,000x in time

2,000x in qubits, 10,000x in time

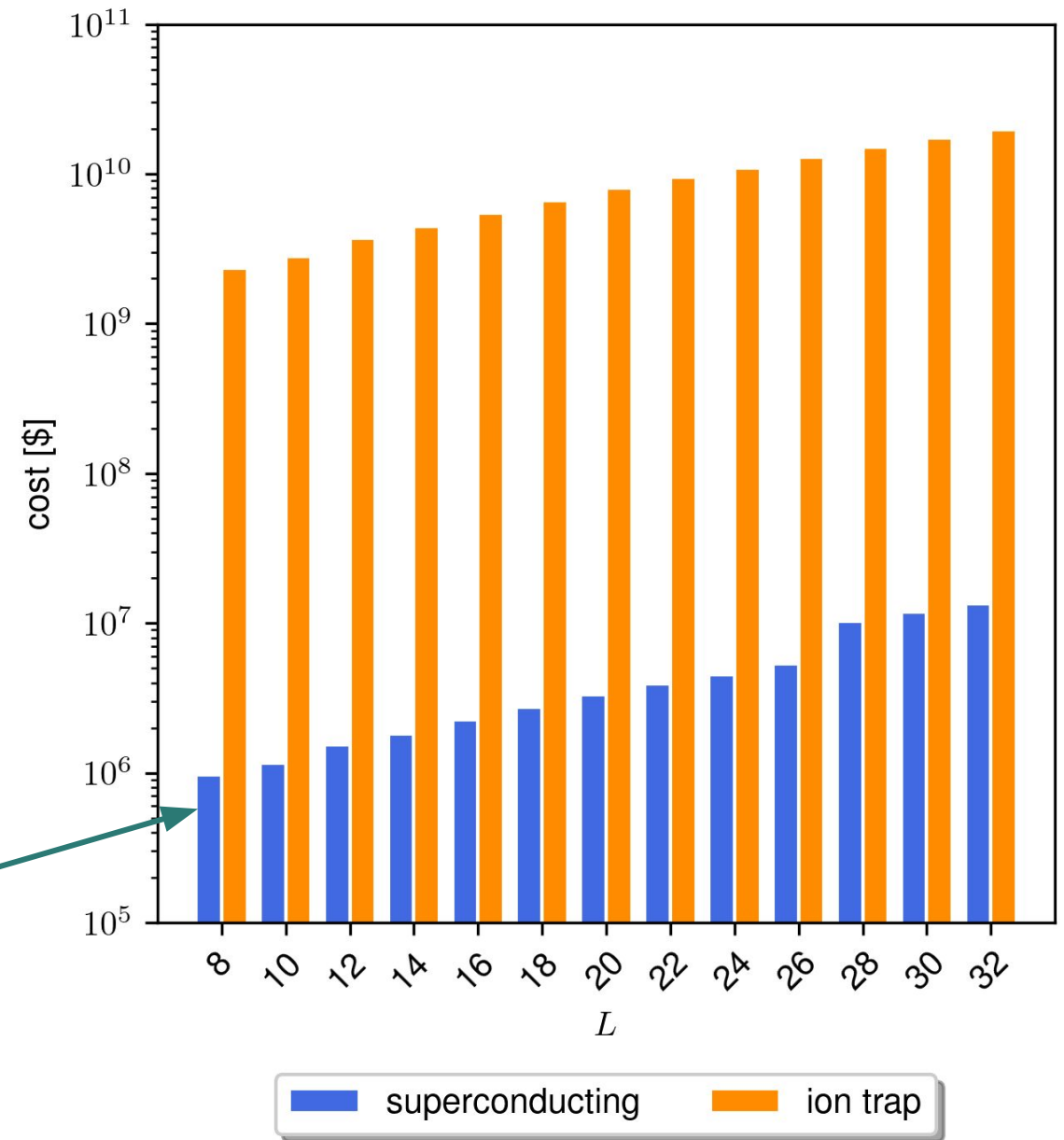
# Flowchart for quantum computing cost estimation



# What does this cost?

Public Pricing data from Quantum Cloud Providers:

Technology	$N_q$	Usage cost [\$/qs]
Superconducting	8-80	0.05
Ion Traps	~20	10





# Conclusions

- Developed new performance model to estimate cost of quantum computation
- Our resource and cost analysis projects that intractable quantum computation could be realized at \$1M - not \$1B!
- Many caveats and uncertainties remain:
  - does pricing model scale linearly?
  - will future QC architectures be modular?
  - improvements in QEC and algorithms?

